





ENERGY ANALYSIS AND MODELING OF MEMBRANE REACTORS PROMECA Workshop 2017







Institute for Micro Process Engineering (IMVT)

Ultra-compact micro structured palladium membrane reactors for hydrogen production

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e Technische Universiteit Eindhoven University of Technology











- 1. Introduction Micro structured membrane reactors for hydrogen production
- 2. On-site steam methane reforming The μ -Enhancer
- 3. Dehydrogenation of liquid organic hydrogen carriers The system MCH / TOL
- 4. Simplified modeling of a micro channel membrane reactor will wall-coated catalyst
 - 1. Derivation of the governing equations
 - 2. Flexible solution using Matlab®
- 5. Some points to remember







SMALL CAPACITY HYDROGEN SUPPLY AN OPPORTUNITY FOR NEW TECHNOLOGY SUCH AS MEMBRANE REACTORS

Customer Hydrogen supply < 500 m³/h on-site - Small quantities (<500m³/h) Transport - low pressure H₂ (<3bar) - medium grade H₂ (>99,95%)

Hydrogen supply < 80.000 m³/h High pressure and purity (~10-20 bar, >99.999%)



Dittmeyer & Schödel, ICCMR-11, 2013





1.0

0.8

700°C

WHY MEMBRANE REACTORS IN HYDROGEN **PRODUCTION?**

- Equilibrium shift •
- Integrated purification •





Η,

Membrane

reformer

NG

more energy-efficient,





BENCHMARK IN MEMBRANE-ASSISTED SMR



Modules:





Yasuda et al., 2005, ICCMR-7, Cetraro





Kurokawa et al., **2010**, Demonstration of Highly-Efficient Distributed Hydrogen Production from Natural Gas with CO₂ Capture, WHEC2010, Essen







Benefits:

- •Very large membrane surface area per catalyst volume (ca. $10^3 10^6 \text{ m}^{-1}$)
- •Negligible mass transport resistance towards membrane even for high-flux membranes
- •Efficient heating by hot gas or catalytic combustion of retentate with air
- •High compactness / low weight / modular plant design







KIT's µ-EnH₂ancer project



⁴⁾ Boeltken et al., Int. J. Hydr. Energy 41 (2014) 18058-18068









Pre-reforming stage

- Cracking of higher hydrocarbons (natural gas)
- Build-up of H₂ partial pressure (by reforming)

Reforming stage

- Reforming
- H₂ separation

Retentate combustion zone

• Heat transfer to the reforming zone





MARIE CURIE ACTIONS

REAL MODULE AND CHANNEL FORMAT



- Modules built from high-temperature corrosion-resistant material (Nicrofer)
- Microchannels by chemical etching (500 x 200 μm)
- Thin palladium foil, i.e., typically 12 µm, sandwiched between two etched microsieves to provide mechanical stability
- Microsieves coated with inorganic diffusion barrier layer to prevent membrane degradation at high temperature
- Catalyst layers by inkjet printing (10-15 µm, 50 mm prereforming, 70 mm reforming/membrane)





SELECTED RESULTS - CONVERSION VERSUS HYDROGEN RECOVERY



Variation of retentate pressure; W/F = 0.33 $g_{Cat}h$ / mol CH₄; S/C = 3

Conversion is close to equilibrium considering the fraction of hydrogen removed



Activity of catalyst high enough to respond to H_2 removal



COMPARISON TO PREVIOUS / OTHER SYSTEMS

| Reference | s _{Pd} [µm] | A _{Pd} [cm ²] | Catalyst | T [K] | ∆p [bar] | X _{CH4} [%] | φ _{H2} [%] | W/F $[g_{Cat}h/mol_{CH_4}]$ | $\begin{array}{c} A_{Pd}/V_{R} \\ [m^{2}/m^{3}] \end{array}$ | \dot{V}_{H_2}/V_R [Nm ² /(m ³ h)] | \dot{V}_{H_2}/A_{Pd} [Nm ² /(m ² h)] |
|--|-------------------------|---------------------------------------|-----------------------------------|-------|------------------|-------------------------|---------------------|--------------------------------|--|---|--|
| Uemiya et al. [4] ¹ | 20 | 25.1 | Ni-based | 773 | 8ª | 90 | 91 | 64.75 | 39.3 | 230 | 5.9 |
| Tong et al. [6] ² | 8 | 20 | Ni/Al ₂ O ₃ | 823 | 10 ^b | 72 | 51 | 74.71 | 110.1 | 1289 | 11.7 |
| Hwang et al. [14] | 6.2 | 33.2 | Ni-based | 813 | 20 | 79 | 96 | n.a. | 6.3 | 58 | 9.2 |
| Tokyo Gas [15,35] ⁴ | 20 | 98918.4 | Ni-based | 823 | 8.7 [°] | 79 | n.a. | n.a. | 8.1 | 33 | 4.1 |
| μ-EnH ₂ ancer [this work] | 12.5 | 26.3 ^d | Rh/Al ₂ O ₃ | 823 | 11 | 87 | 92 | 0.33 | 41/66 ^e | 472/739 ^e | 12.1 |
| ^a Sweep gas 500 ml/min Ar ($p_{H_2,Perm} = 0.38$ bar). ^b Sweep gas 470 ml/min N ₂ . ^c Vacuum ($p_{H_2,Perm} = 0.4$ bar). ^d Effective membrane area (across the microchannels). ^e Optimized membrane module with thinner microstructured foils. | | | | | | | | | | | |
| | | | | | | | | | | | |
| Fernandez 5) | 3-5 | 160 | Ni-based | 823 | 1.27 | 76 | 20 | n.a. | 8.0 | 21 | 3.0 |
| Mahecho- Botero ର | 25 | 1800 | Ni-based | 823 | 8.97 | 80 | 62.5 | n.a. | 22.5 | 125 | 5.5 |

- Highest hydrogen production rate per membrane area
- High volumetric hydrogen production rate
- Very compact 41/66 m⁻¹

Boeltken et al., Int. J. Hydr. Energy, **2014**, 39, 18058-18068.

1) Uemiya et al., Appl. Catal. **1991**, 67, 223 - 230.

2) Tong et al., Catal. Today **2006**, 111, 147 - 152.

- 3) Hwang et al., Int. J. Hydr. Energy **2012**, 37, 6601 6607.
- 4) Shirasaki et al., Int. J.Hydr. Energy **2009**, 34, 4482 4487.
- 5) Fernandez et al., Int. J. Hydr. Energy **2017**, 42, 13763-13776.

6) Mahecha-Botero et al., Chem. Eng. Sci. 2008, 63, 2752-2762.







NEW SYSTEM WITH FLUIDIC HEATING ESTABLISHED



Dittmeyer et al., ICIM-14 2016





Combustion module 5 plates, 7.2 mm height



Gas distribution

Combustion fuel feeding

Combustion fuel distribution into plate 4

Oxidant gas feeding and combustion

Gas distribution



Reforming module 8 plates; 9.4 mm height



Gas distribution Gas distribution Hydrogen removal Etched micro sieve support Palladium foil Etched micro sieve support Reforming zone Pre-reforming zone Gas distribution

- all plates fabricated
- microchannel plates awaiting coating with new catalyst
- new porous metal-supported membranes in preparation

NEW MODULES

Module with integrated reforming and combustion zones 11 plates; 14.4 mm height



Gas distribution

Combustion fuel feeding and distribution into plate 3

Oxidant gas feeding and combustion

Gas distribution

Gas distribution

Permeate gas

Etched micro sieve support

Palladium foil

Etched micro sieve support

Reforming zone

Pre-reforming zone

Gas distribution







IMRET 2018, OCT. 21-24, KARLSRUHE

GENERAL INFORMATION

EXHIBITION

Are you a technology or solution provider and would like to actively contribute to this conference?

The exhibition is an integral part of the IMRET and offers a unique opportunity to promote your technologies, products and services. Equipment suppliers, engineering companies as well as research institutes are invited to participate in the exhibition.



VENUE

Gartenhalle Karlsruhe Festplatz 3 76137 Karlsruhe / Germany

ORGANISER

DECHEMA e.V. Theodor-Heuss-Allee 25 60486 Frankfurt am Main / Germany www.dechema.de

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DECHEMA Gesellschaft für Chemische Techn und Biotechnologie e.M.

FIRST ANNOUNCEMENT

21 – 24 October 2018 Karlsruhe · Germany

IMRET 2018 15th International Conference on Micro Reaction Technology

www.dechema.de/IMRET2018





SUPPORTED BY: ProcessNet Working Group Microreaction Engineering and International Flow Chemistry Society

INVITATION TO IMRET 15

For its 15th edition, the International Conference on Micro Reaction Technology (IMRET) is coming back to where it all began. Since the start of this "place to be" for chemists, chemical engineers and micro systems engineers, the event has demonstrated a variety of new applications in chemical engineering, chemistry, related disciplines like energy supply, and even more different fields such as analytics, life science and medicine. Yet, as each of the involved disciplines has seen amazing progress, the mission of providing an effective platform for exchange is more topical than ever.

IMRET 15 therefore sets out to become once more the place where new developments regarding all facets of micro process engineering and flow chemistry will be jointly discussed for mutual benefit.

IMRET 15 will take place in the city of Karlsruhe from 21 October to 24 October 2018, organised by DECHEMA e.V. with the support of the German ProcessNet Working Group Microreaction Engineering, the IMRET Steering Committee and the International Flow Chemistry Society.



TOPICS / DATES / COMMITTEE

CONFERENCE TOPICS

- » Mixing and heat transfer in micro systems
- » Reactions and catalysis in flow systems
- » Downstream processing
- » Modelling/simulation
- » Fabrication of micro-structured devices
- » Process automation, sensors, digitalization
- » Modular plant concepts
- » New applications in chemistry, biology, energy etc.
- » Industrial implementation

IMPORTANT DATES

| October 2017 | Submission of abstract is open | | | |
|----------------------|--------------------------------|--|--|--|
| February 2018 | End of abstract submission | | | |
| April 2018 | Notification of acceptance | | | |
| May 2018 | Final programme | | | |
| 21 - 24 October 2018 | IMRET 15 takes place | | | |

LOCAL ORGANISING COMMITTEE

| DECHEMA e.V. |
|--------------------------------------|
| KIT, Institute for Micro Process |
| Engineering |
| Max Planck Institute of Colloids and |
| Interfaces |
| BASF SE |
| Fraunhofer ICT-IMM |
| Fraunhofer Institute for Chemical |
| Technology ICT |
| |

www.dechema.de/IMRET2018







RH/AL₂O₃ CATALYSTS BY FLAME SPRAY PYROLYSIS

| | Rh loading ^a (%) | H ₂ chemis | sorption re | TEM results | | |
|-----------|-----------------------------------|--|--|---|--|---|
| Catalyst | | Rh active sites $(A_{chem})^b$ $(\mu mol/g_{cat})$ | Rh dis- persion ^c (%) | Rh particle size (P _{chem}) ^d (nm) | Rh particle size (P _{TEM}) ^e (nm) | Rh active sites (A _{TEM}) ^f (µmol/g _{cat}) |
| 5Rh/FA1 | 3.85 | 174 | 47 | 2.4 | 1.6 | 290 |
| 5Rh/SA1 | 2.75 | 117 | 44 | 2.5 | 3.5 | 90 |
| 1Rh/FA1 | 0.94 | 69 | 75 | 1.5 | 1.0 | 105 |
| 1Rh/SA1 | 0.84 | 59 | 73 | 1.5 | 1.7 | 53 |
| 0.2Rh/FA1 | 0.20 | 13 | 69 | 1.6 | _ | |
| 0.2Rh/SAl | 0.20 | 12 | 61 | 1.8 | _ | |



Yu et al., Appl. Catal. B: Environ. 2016, 198, 171–179.

- a Rh loading derived from ICP-OES.
- b From H_2 uptake in chemisorption assuming a stoichiometry of H/Rh = 1.
- c Ratio of active Rh from H_2 chemisorption and Rh content from ICP-OES.
- d From metal dispersion by H_2 chemisorption.
- e From TEM measurements.
- f Derived from TEM results.



Temperature °C



In cooperation with Forschungszentrum Jülich (Martin Bram, IEK-1)



- Pd: ca. 4 12 μm (foil or SPS coating)
- 8-YSZ: ca. 20 40 µm
- Sinter metal: ca. 1 mm

Dittmeyer et al., ICCMR-13 2017

Test specimen

Testing unit







Boeltken et al., CE&P 2013, 67,136-147

0



POROUS SHEETS FROM CROFER 22 APU BY TAPE CASTING / SINTERING

Forschungszentrum Jülich (Martin Bram, IEK-1)



- Porosity: $27 \pm 4\% (29.5 \pm 0.9\%)$
- Thickness: 1.08 \pm 0.05 mm



- N₂ permeability: 0.162 \pm 0.003 µmol/m/s/Pa
- N_2 permeance: 1.5×10^{-4} mol/m²/s/Pa



COATING WITH POROUS 8YSZ

Forschungszentrum Jülich (Martin Bram, IEK-1, Paul Kant, IMVT)







SINGLE VERSUS DOUBLE LAYER COATING

Forschungszentrum Jülich (Martin Bram, IEK-1, Paul Kant, IMVT)



Layer thickness derived from SEM (Matlab® routine)



DOUBLE LAYER COATING

Forschungszentrum Jülich (Martin Bram, IEK-1, Paul Kant, IMVT)





DOUBLE LAYER COATING

Forschungszentrum Jülich (Martin Bram, IEK-1, Paul Kant, IMVT)



Nice smooth surface for coating with thin Pd or Pd alloy layer



SUSPENSION PLASMA SPRAYING OF PD NANOPARTICLES

In cooperation with the German Aerospace Center (Sayed-Asif Ansar, Dirk Ullmer, TT, Stuttgart)



- Stable suspension of Pd nanoparticles obtained
- Injection system and plasma parameters being optimised
- First coating experiments performed on test specimen (Pd layers are not yet gastight)
- Transfer to membrane reformer modules















Influence of pressure on the deactivation rate.

T = 325° C. modified contact time W/F 4000 kg s m⁻³ (1 bar) and 750 kg s m⁻³ (9 bar). MCH/N₂ 50/50.



 Kinetic studies in BERTY-type recycle reactor showed much slower deactivation by carbon formation

Kreuder et al., Catal. Today, 2015, 242, 211-220.







Micro Packed-Bed Membrane Reactor – First Design

| | Packed bed P | Packed-bed and permeate sections | | | | |
|---|--|----------------------------------|--|--|--|--|
| | | microchannels Pd-membrane | etal foil th holes Catalyst | | | |
| height 2 mm length 20 mm width 5 mm | | 2 500 (JM) 200 (| 100 um 1004 | | | |
| 100,00 | | 150 µm | 100 µm 150 | | | |
| Module | $Q_0 \text{ mol}^*\text{m}^{-1*}\text{s}^{-1*}\text{Pa}^{-0.5}$ | $E_a kJ^*mol^{-1}$ | Q (350 °C) mol*m ⁻¹ *s ⁻¹ *Pa ^{-0.5} | | | |
| membrane B membrane C membrane D | 2.8*10 ⁻⁷ 2.0*10 ⁻⁶ 3.7*10 ⁻⁷ | 14.6 26.7 16.8 | 1.7*10 ⁻⁸ 1.1*10 ⁻⁸ 1.5*10 ⁻⁸ | | | |
| Boeltken et al. [18] | $2.6^{*}10^{-7}$ | 14.6 | 1.6^*10^{-8} | | | |

Kreuder et al., Int. J. Hydr. Energy, **2016**, 41, 12082-12092.

S-

>3000 >25,000 >85,000 >30,000



CATALYST DEACTIVATION BY COKING

fresh

Effect of hydrogen on deactivation. $W/F = 250 \text{ kg s m}^{-3}$.



350° C, 9 bar back permeation of H_2 in entrance region

400°C, 9 bar

 348° C, 1 bar perfect back mixing, i.e., H₂ concentration at reactor effluent level

no back mixing, i.e., low H₂ partial

pressure in entrance region

after use

Carbon content stable around 0.8 wt.-% (TGA, 400° C, BERTY reactor, up to 25 h)



REGNERATION IN PLACE

Procedure

- 1 h treatment in a flow of 5 ml/min air plus 50 ml/min N₂ for during off the deposits
- 1 h treatment in a flow of 50 ml/min H_2 for reduction of the catalyst surface



Kreuder et al., Int. J. Hydr. Energy, 2016, 41, 12082-12092.



Reduced bed height of 0.5 mm / enlarged area



On-going work:

- Further optimisation of reactor geometry based on simulations (optimised bed height, longer packed bed section).
- Catalyst improvement regarding coking.
- Scale-up and test in integrated process.

Kreuder et al., Int. J. Hydr. Energy, 2016, 41, 12082-12092.











new flow velocity u(x+dx) for ideal gas and constant pressure:

(x+dx) for ideal gas
ure:

$$u(x + dx) \cdot w \cdot h = u(x) \cdot w \cdot h + \left[\sum_{i} k_{ig} \cdot \left(c_{i}^{*}(x + dx) - c_{i}(x + dx)\right)\right] \cdot w \cdot dx \cdot \frac{RT}{p}$$

total molar flux to/from layer

$$u(x+dx) = u(x) + \frac{dx}{h} \cdot \frac{RT}{p} \sum_{i} k_{ig} \cdot \left(c_i^*(x+dx) - c_i(x+dx)\right)$$

in material balance:

$$0 = u(x) \cdot \left[c_i(x) - c_i(x + dx) \right] - \frac{dx}{h} \cdot \frac{RT}{p} \cdot \left[\sum_i k_{ig} \cdot \left(c_i^*(x + dx) - c_i(x + dx) \right) \right] \cdot c_i(x + dx) + \dots \\ \dots k_{ig} \cdot \left(c_i^*(x + dx) - c_i(x + dx) \right) \cdot \frac{dx}{h}$$

concentration on the surface is given

by reaction/diffusion inside layer

032





DOUBLE LAYER WITH ADJACENT PD MEMBRANE

now, the situation with Pd membrane:









same approach:

CSTR material balance component i (reaction side):

$$0 = q(x) \cdot c_i(x) - q(x + dx) \cdot c_i(x + dx) + j_i(x + dx) \cdot A + j_{i,M}(x + dx) \cdot A_M$$

$$0 = \left[u(x) \cdot c_i(x) - u(x + dx) \cdot c_i(x + dx) \right] \cdot w \cdot h + k_{ig} \cdot \left(c_i^*(x + dx) - c_i(x + dx) \right) \cdot w \cdot dx + \dots$$

$$\dots j_{i,M}(x + dx) \cdot w \cdot f_M \cdot dx$$



the membrane flux connects both compartments:

$$j_{i,M}(x+dx) = -\frac{Q_{H_2}}{S_M} \cdot (RT)^{0.5} \cdot (C_{H_2}^{0.5}(x+dx) - C_{H_2,p}^{0.5}(x+dx)) \quad \text{for } i = H_2$$

$$j_{i,M}(x+dx) = 0 \quad \text{for } i \neq H_2$$

here $j_{i,M}$ is positiv if hydrogen enters the reaction compartment via the membrane



again, new flow velocity u(x+dx) for ideal gas and constant pressure:

$$u(x + dx) \cdot w \cdot h = u(x) \cdot w \cdot h + \left[\sum_{i} k_{ig} \cdot \left(c_i^*(x + dx) - c_i(x + dx)\right)\right] \cdot w \cdot dx \cdot \frac{RT}{p} + \dots$$

total molar flux to/from layer
total molar flux to/from permeate

$$\dots - \frac{Q_{H_2}}{s_M} \cdot (RT)^{0.5} \cdot (C_{H_2}^{0.5}(x + dx) - C_{H_2,p}^{0.5}(x + dx)) \cdot W \cdot f_M \cdot dx \cdot \frac{RT}{p}$$

total volume flow to/from permeate

total volume flow to/from layer

$$u(x + dx) = u(x) + \frac{dx}{h} \cdot \frac{RT}{\rho} \cdot \left[\sum_{i} k_{ig} \cdot (c_{i}^{*}(x + dx) - c_{i}(x + dx)) - ... \right]$$
$$...f_{M} \cdot \frac{Q_{H_{2}}}{S_{M}} \cdot (RT)^{0.5} \cdot (c_{H_{2}}^{0.5}(x + dx) - c_{H_{2},\rho}^{0.5}(x + dx)) \right]$$





FREE CHANNEL VOLUME (REACTION SIDE)

in material balance:

concentration on the surface is given by reaction/diffusion inside layer

concentration in the permeate (has to be determined by permeate side material balance)







here as well, new flow velocity $u_p(x+dx)$ for ideal gas and constant pressure:

$$u_{p}(x+dx) \cdot w_{p} \cdot h_{p} = u_{p}(x) \cdot w_{p} \cdot h_{p} + \dots$$

$$\dots \frac{Q_{H_{2}}}{S_{M}} \cdot (RT)^{0.5} \cdot (C_{H_{2}}^{0.5}(x+dx) - C_{H_{2},p}^{0.5}(x+dx)) \cdot w \cdot f_{M} \cdot dx \cdot \frac{RT}{p}$$
total molar flux to/from permeate
$$(total volume flow to/from permeate)$$

$$u_{p}(x+dx) = u_{p}(x) + \frac{dx}{h_{p}} \cdot f_{M} \cdot \frac{w}{w_{p}} \cdot \frac{(RT)^{1.5}}{p} \cdot \frac{Q_{H_{2}}}{S_{M}} \cdot \left[C_{H_{2}}^{0.5}(x+dx) - C_{H_{2},p}^{0.5}(x+dx)\right]$$

note that pressure p here is the permeate pressure, which is in general different from the reaction side pressure



in material balance:

$$0 = \boldsymbol{u}_{p}(\boldsymbol{x}) \cdot \left[\boldsymbol{c}_{i,p}(\boldsymbol{x}) - \boldsymbol{c}_{i,p}(\boldsymbol{x} + d\boldsymbol{x})\right] + \dots$$
$$\dots \frac{d\boldsymbol{x}}{h_{p}} \cdot \boldsymbol{f}_{M} \cdot \frac{\boldsymbol{w}}{\boldsymbol{w}_{p}} \cdot \frac{(RT)^{1.5}}{p} \cdot \frac{\boldsymbol{Q}_{H_{2}}}{\boldsymbol{s}_{M}} \cdot \left(\boldsymbol{c}_{H_{2}}^{0.5}(\boldsymbol{x} + d\boldsymbol{x}) - \boldsymbol{c}_{H_{2},p}^{0.5}(\boldsymbol{x} + d\boldsymbol{x})\right) \cdot \boldsymbol{c}_{i,p}(\boldsymbol{x} + d\boldsymbol{x}) - \dots$$
$$\dots j_{i,M} \cdot \boldsymbol{f}_{M} \cdot \frac{d\boldsymbol{x}}{h_{p}}$$

again, note that pressure p here is the permeate pressure, which is in general different from the reaction side pressure





- different diffusivities D_i in the two layers can be handled via position-dependent effective diffusivity $D_i(y)$
- the presence of different catalysts in the two layers can also be handled via position-dependent catalyst mass concentrations ρ_{Cat,j}(y)





Effective diffusivity as step function:

$$D_i = D_{i,\text{Core}} + f(y) \cdot \left(D_{i,\text{Shell}} - D_{i,\text{Core}} \right)$$

Catalyst mass concentrations as step functions:

 $\rho_{Cat,Core}(\mathbf{y}) = [1 - f(\mathbf{y})] \cdot \rho_{Core}$ $\rho_{Cat,Shell}(\mathbf{y}) = f(\mathbf{y}) \cdot \rho_{Shell}$



influx

ODE SYSTEM FOR POSITION-DEPENDENT DIFFUSIVITY $D_{I}(Y)$

$$0 = -D_i(y) \cdot \frac{dc_i}{dy}(y) \cdot dx \cdot w + D_i(y + \Delta y) \cdot \frac{dc_i}{dy}(y + \Delta y) \cdot dx \cdot w + \left[\frac{R_i(y) + R_i(y + \Delta y)}{2}\right] \cdot \Delta y \cdot dx \cdot w$$

outflux

source term

$$0 = -D_{i} \cdot \frac{dC_{i}}{dy} + \left[D_{i} + \frac{dD_{i}}{dy} \cdot \Delta y\right] \cdot \left[\frac{dC_{i}}{dy} + \frac{d^{2}C_{i}}{dy^{2}} \cdot \Delta y\right] + \left[\frac{R_{i} + R_{j} + \frac{dR_{j}}{dy} \cdot \Delta y}{2}\right] \cdot \Delta y$$

$$0 = -D_{i} \cdot \frac{dC_{i}}{dy} + \left[D_{i} \cdot \frac{dC_{i}}{dy} + \frac{dD_{i}}{dy} \cdot \Delta y \cdot \frac{dC_{i}}{dy} + D_{i} \cdot \frac{d^{2}C_{i}}{dy^{2}} \cdot \Delta y + \frac{dD_{i}}{dy} \cdot \frac{d^{2}C_{i}}{dy^{2}} \cdot (\Delta y)^{2}\right] + \left[R_{i} \cdot \Delta y + \frac{1}{2} \cdot \frac{dR_{i}}{dy} \cdot (\Delta y)^{2}\right]$$





$$\frac{d^2 c_i}{dy^2} = -\frac{R_i(y)}{D_i(y)} - \frac{1}{D_i(y)} \cdot \frac{dD_i(y)}{dy} \cdot \frac{dc_i}{dy}$$

...

where:

$$\frac{dD_i(y)}{dy} = \left(D_{i,\text{core}} - D_{i,\text{shell}}\right) \cdot \frac{df}{dy}$$

and:
$$\frac{df}{dy} = \frac{df}{dy' \cdot s} = \frac{1}{s} \cdot \left(1 - \tanh^2 \left[\left(y' - y'_0 \right) \cdot 2\pi \cdot f_s \right] \right) \cdot \pi \cdot f_s$$
$$\int \int \frac{dy}{dy' \cdot s} = \frac{1}{s} \cdot \left(1 - \tanh^2 \left[\left(y' - y'_0 \right) \cdot 2\pi \cdot f_s \right] \right) \cdot \pi \cdot f_s$$
due to:
$$s = h_{core} + h_{shell}$$
$$y' = \frac{y}{s}$$
$$y'_0 = \frac{h_{core}}{s}$$





$$\frac{d^2 c_i}{dy^2} = -\frac{R_i(y)}{D_i(y)} - \frac{1}{D_i(y)} \cdot \frac{dD_i(y)}{dy} \cdot \frac{dc_i}{dy}$$

Boundary conditions:





...

Case 1: infinite mass transfer rate

$$\mathbf{C}_{i}\big|_{y=h_{\text{Core}}+h_{\text{Shell}}}=\mathbf{C}_{i}\left(\mathbf{X}+\mathbf{d}\mathbf{X}\right)$$

flux from/to layer is expressed with the concentration gradient at top of the double layer

$$0 = u(x) \cdot \left[c_i(x) - c_i(x + dx) \right] - \frac{dx}{h} \cdot \frac{RT}{p} \cdot \left[\sum_i k_{ig} \cdot \left(c_i^*(x + dx) - c_i(x + dx) \right) - c_i(x + dx) \right) \right]$$

... $f_M \cdot \frac{Q_{H_2}}{S_M} \cdot \left(RT \right)^{0.5} \cdot \left(c_{H_2}^{0.5}(x + dx) - c_{H_2,p}^{0.5}(x + dx) \right) \right] \cdot c_i(x + dx) + \dots$
... $k_{ig} \cdot \left(c_i^*(x + dx) - c_i(x + dx) \right) \cdot \frac{dx}{h} + j_{i,M} \cdot f_M \cdot \frac{dx}{h}$

where:
$$k_{ig} \cdot \left(c_i^* (x + dx) - c_i (x + dx) \right) = -D_{i,Shell} \cdot \frac{dc_i}{dy} \Big|_{y = h_{Core} + h_{Shell}}$$



EVALUATION

$$0 = u(x) \cdot \left[C_{i}(x) - C_{i}(x + dx) \right] - \frac{dx}{h} \cdot \frac{RT}{p} \cdot \left[\sum_{i} -D_{i,\text{Shell}} \cdot \frac{dC_{i}}{dy} \right]_{y = h_{\text{Core}} + h_{\text{Shell}}} - \dots$$
$$\dots f_{M} \cdot \frac{Q_{H_{2}}}{S_{M}} \cdot \left(RT \right)^{0.5} \cdot \left(C_{H_{2}}^{0.5}(x + dx) - C_{H_{2},p}^{0.5}(x + dx) \right) \right] \cdot C_{i}(x + dx) - \dots$$
$$\dots D_{i,\text{Shell}} \cdot \frac{dC_{i}}{dy} \Big|_{y = h_{\text{Core}} + h_{\text{Shell}}} \cdot \frac{dx}{h} + j_{i,M} \cdot f_{M} \cdot \frac{dx}{h}$$

rearranging:

$$u(x) \cdot c_{i}(x) - \frac{dx}{h} \cdot \left[D_{i,Shell} \cdot \frac{dc_{i}}{dy} \right]_{y=h_{Care}+h_{Shell}} - j_{i,M} \cdot f_{M}$$

$$c_{i}|_{y=h_{Care}+h_{Shell}} = c_{i}(x+dx) = \frac{u(x) - \frac{dx}{h} \cdot \frac{RT}{p} \cdot \left[\sum_{i} D_{i,Shell} \cdot \frac{dc_{i}}{dy} \right]_{y=h_{Care}+h_{Shell}} + f_{M} \cdot \frac{Q_{H_{2}}}{s_{M}} \cdot (RT)^{0.5} \cdot (c_{H_{2}}^{0.5}(x+dx) - c_{H_{2},p}^{0.5}(x+dx))$$

- c_{H2,p}(x+dx) is found from the hydrogen material balance for permeate side for given sweep gas flow rate, permeate pressure and reaction side hydrogen concentration.
- owing to Sievert's law this requires the solution of a nonlinear equation.



Flexible approach - 1D cascade of cells with the option to limit the concentration change per cell via step size control

- solved profiles for one cell are used as initial guess for subsequent cell
- mole flows in both compartments are updated based on solved profiles
- graphics for monitoring progress of the calculation
- material balance checks
- heat balance not yet implemented
- pressure drop along channel neglected

bvp4c - reliable boundary value problem solver with adaptive grid

- CSTR material balance and membrane transport integrated in the definition of the boundary conditions
- nonuniform catalyst distribution and effective diffusivity two distinct layers approximated by S-shaped distribution function

Exchangeable kinetics and permeation









First commercial applications of membrane reactors may appear in

small-capacity hydrogen production for industrial uses via on site reforming (low pressure, moderate purity),
hydrogen generation from LOHC in the context of hydrogen logistics,

rather than in large-scale reforming or WGS.

•Transport effects may have a big influence on reactor performance (yield, selectivity, space time yield, etc.); this holds especially for membrane reactors where the reactions kinetics should not only match the usual heat and mass transport rates but also the permeation kinetics.

•Matlab is a flexible platform for building your own customised models for "multiscale" reactor simulation



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